

Discounting The Future - Part I

Valuing A Term Bond

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In this white paper we will value a term bond. To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

We are tasked with valuing a term bond. The table below presents our bond model parameters...

Description	Value
Bond face (principal) value	\$100,000
Bond term in years	5
Number of annual coupon pmts	1
Annual bond coupon rate	4.75 %
Annual real rate of return	2.25 %
Annual inflation rate	2.75 %
Annual risk premium	1.70 %

The bond pays one coupon payment annually. We will define the variable C to be that coupon payment amount in dollars. Using the table above, the coupon payment calculation is...

$$C = \text{Bond principal} \times \text{Coupon rate} = \$100,000 \times 0.0475 = \$4,750 \quad (1)$$

We will define the variable k to be the annual market interest rate. The market interest rate is the interest rate that the market demands to buy this bond. Using the table above, the market rate calculation is...

$$k = \text{Real rate of return} + \text{Inflation rate} + \text{Risk premium} = 0.0225 + 0.0275 + 0.0170 = 0.0670 \quad (2)$$

Building Our Model

We will define the variable V_0 to be the market value of our bond at time zero. The equation for bond value is...

$$V_0 = \text{Value of bond coupon payments} + \text{Value of bond face value} \quad (3)$$

We will define the variable F to be bond face value and the variable α to be the bond coupon rate. The equation for bond market value at time zero is...

$$V_0 = \frac{C}{(1+k)^1} + \frac{C}{(1+k)^2} + \frac{C}{(1+k)^3} + \frac{C}{(1+k)^4} + \frac{C+F}{(1+k)^5} \quad \dots \text{where... } C = \alpha F \quad (4)$$

We will define the variable t to be time in years. Note that we can rewrite Equation (4) above as...

$$V_0 = \sum_{t=1}^5 \frac{C}{(1+k)^t} + \frac{F}{(1+k)^5} \quad (5)$$

We will define the variable θ to be the discount factor. Using this definition, we can rewrite Equation (5) above as...

$$V_0 = C \sum_{t=1}^5 \theta^t + F \theta^5 \quad \dots \text{where... } \theta = \frac{1}{1+k} \quad (6)$$

Note the solution to the following geometric series... [1]

$$\sum_{t=1}^5 \theta^t = \frac{\theta - \theta^{(t-1)}}{1 - \theta} \dots \text{given that... } 0 < \theta < 1 \quad (7)$$

The Answer To Our Hypothetical Problem

Using Equations (2) and (7) above, the equation for the value of our discount factor is...

$$\theta = \frac{1}{1 + k} = \frac{1}{1 + 0.0670} = 0.93721 \quad (8)$$

Using Equations (6) and (8) above, the value of our bond coupon payments is...

$$\text{Value of coupon payments} = 4,750 \times \frac{0.93721 - 0.93721^6}{1 - 0.93721} = 19,633.39 \quad (9)$$

Using Equations (6) and (8) above, the value of our bond principal payments is...

$$\text{Value of principal payments} = 100,000 \times \theta^5 = 72,306.59 \quad (10)$$

Using Equations (3), (9) and (10) above, the value of our term bond is...

$$V_0 = \text{Bond market price} = 19,633.39 + 72,306.59 = 91,939.98 \quad (11)$$

References

[1] Wikipedia - List of mathematical series